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# Milgrom's Revision of Cosmic Dynamics: Amending Newton's Laws or Kepler's?

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MILGROM'S REVISION OF COSMIC DYNAMICS:

AMENDING NEWTON'S LAWS OR KEPLER'S?

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ABSTRACT

Milgrom's recent revision of Newtonian dynamics was introduced to eliminate the inference that large quantities of invisible mass exist in galaxies. I show by simple examples that a Milgrom acceleration, in the form presented so far, implies other far-reaching changes in dynamics. The momentum of an isolated system is not conserved, and the usual theorem for center-of-mass motion of any system does not hold. Naive applications require extreme caution. The model fails to provide a complete description of particle dynamics and should be thought of as a revision of Kepler's laws rather than Newton's.

The Milgrom acceleration also implies fundamental changes in cosmology. A quasi-Newtonian calculation adapted from Newtonian cosmology suggests that a "Milgrom universe" will recollapse even if the classical closure parameter  $\Omega$  is  $\ll 1$ . The solution, however, fails to satisfy the cosmological principles. I examine reasons for the breakdown of this calculation. A new theory of gravitation will be needed before the behavior of a Milgrom universe can be predicted.

Subject headings: cosmology — galaxies: clusters of — galaxies: internal motions -- stars: stellar dynamics

## I. MILGROM ACCELERATION

The astronomical evidence (Faber and Gallagher 1979; Davis et al. 1980) for large amounts of "invisible mass" (essentially, mass not contained in luminous stars) in and around galaxies comes almost entirely from applications of Newton's second law to galaxies and galaxy systems. The accelerations in these systems are much smaller than those for which the law has been tested in the laboratory or in the solar system.

In three heretical papers, Milgrom (1983a, b, c) has proposed to do away with invisible mass by altering the second law. Inter alia, he proposes that, at least with respect to gravitational forces, Newton's  $\underline{a} = \underline{F}_N/\underline{m}$  should be replaced, in the low-acceleration limit, by

$$\underline{a} \approx \sqrt{\left(\underline{a}_0 \frac{\underline{F}_N}{\underline{m}}\right)} \equiv \sqrt{(\underline{a}_0 \underline{g}_N)}. \quad (1)$$

Here  $\underline{g}_N$  is the Newtonian acceleration calculated from the mass distribution in the usual way, and  $\underline{a}_0$  is a new physical constant having dimensions of acceleration. Law (1) is assumed to apply when the true acceleration  $\underline{a}$  is  $\ll \underline{a}_0$ . Milgrom finds that he can explain the flat rotation curves of galaxies and large virial velocities in clusters without adding invisible mass. By equation (1), small accelerations due to a given galaxy mass are larger than they would be in Newtonian theory. He can also avoid smaller-scale observational limits on non-Newtonian forces, e.g., from solar-system observations. All this is possible provided

$$\underline{a}_0 \approx 8 \times 10^{-8} h^2 \text{ cm sec}^{-2} \approx \left(\frac{4}{5} h\right) c H_0, \quad (2)$$

where  $H_0$  is the Hubble constant and  $h$  is the dimensionless Hubble constant  $H_0/(100 \text{ km sec Mpc}^{-1})$ , of order unity. The numerical value  $8 \times 10^{-8}$  in

equation (2) is roughly equal to the Newtonian acceleration occurring in the inner parts of a galaxy. In the outer parts, then,  $\underline{a} < \underline{a}_0$ , and Milgrom's law (1) comes into play.

The relation (2) to  $\underline{c}H_0$  is interesting. A "cosmic acceleration"  $\underline{c}H_0$  would reduce a speed  $\underline{c}$  to speed zero in a Hubble time. From Milgrom's point of view, relation (2) is fortuitous and adds one more to the list of "numerical coincidences" in cosmology (Bondi 1960, § 7.1). It suggests vaguely a Machian basis for the Milgrom acceleration and the constant  $\underline{a}_0$  and is therefore regarded as an asset to the model.

## II. DYNAMICAL PROBLEMS

Without dismissing Milgrom's ideas out of hand, I wish to point out that a dynamical law of type (1) has other consequences (some undesirable) in addition to the consequences sought by Milgrom. Note at once that the accelerations  $\underline{a}$  given to a test particle by two or more attracting bodies acting jointly do not add linearly; the Newtonian accelerations  $\underline{g}_N$  do add linearly, so their square roots cannot do so. Next consider the dynamics of some simple multiparticle systems. Assume that equation (1) gives the correct dynamics for particles with low acceleration. Consider a system consisting of two particles only, with masses  $\underline{m}_1$  and  $\underline{m}_2$ , interacting gravitationally. Let them be placed at rest on the  $\underline{x}$  axis, with  $\underline{x}_2 - \underline{x}_1 = \underline{r} > 0$ . Let  $\underline{m}_1, \underline{m}_2$  be small enough so that equation (1) applies. Set  $\underline{F}_N = \underline{G}\underline{m}_1\underline{m}_2/\underline{r}^2$ . Note that the masses are constant, for law (1) is intended to apply only in the nonrelativistic limit. Differentiating the total momentum  $\underline{p} \equiv \underline{p}_1 + \underline{p}_2$  and using equation (1), we find

$$\dot{p} \equiv \frac{dp}{dt} = (a_0 F_N)^{1/2} (m_1^{1/2} - m_2^{1/2}) . \quad (3)$$

When  $m_1 \neq m_2$ ,  $p$  for this isolated system is not conserved. This is obvious from equation (1), because the two accelerations are not inversely proportional to the masses as in Newtonian dynamics. Except for the special case  $m_1 \approx m_2$ ,  $|\dot{p}|$  is of the same order as the larger of  $|\dot{p}_1|$  and  $|\dot{p}_2|$ .

Consider a second simple case: a system  $S$  consisting of two particles  $m_1$  and  $m_2$ ,  $S$  being placed in the gravitational field of a third and larger body  $m_3$ . Place these objects at rest on the  $x$  axis, with  $x_2 - x_1 \equiv r > 0$  and  $x_3 - x_2 \equiv R > 0$ . Let  $m_1$  and  $m_2$  be small enough so that the interaction of  $m_1$  and  $m_2$  can be neglected, and let  $R$  be large enough so that law (1) describes the motion of  $m_1$  and  $m_2$  in the field of  $m_3$ ; i. e., the accelerations of  $m_1$  and  $m_2$  respectively in the field of  $m_3$  are given by equation (1), with

$$\frac{F_N}{m} = \frac{F_{Ni}}{m_i} = g_{Ni} \quad (i = 1, 2) , \quad (4)$$

where  $g_{Ni}$  is the Newtonian acceleration produced by  $m_3$  at particle  $i$ . The center of mass of  $S$  is

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} . \quad (5)$$

Differentiating and using equation (1), we find that the acceleration of the center of mass is

$$\ddot{x}_{CM} = \frac{\sqrt{a_0}}{m_1 + m_2} (m_1 \sqrt{g_{N1}} + m_2 \sqrt{g_{N2}}) . \quad (6)$$

Let us ask whether the usual Newtonian theorem on motion of the center of mass is valid for Milgrom accelerations. The external forces on system  $S$  are  $\underline{F}_{N1}$  and  $\underline{F}_{N2}$ . Therefore the center-of-mass theorem applied to law (1) would say

$$\ddot{x}_{CM} = \sqrt{\left(a_0 \frac{F_{N1} + F_{N2}}{m_1 + m_2}\right)} = \sqrt{\left(\frac{a_0}{m_1 + m_2}\right)} \sqrt{(m_1 g_{N1} + m_2 g_{N2})} \quad (7)$$

A little algebra shows that expressions (6) and (7) are equal only if  $g_{N1} = g_{N2}$ . In general, then, if individual test particles in an external gravitational field obey law (1), their center of mass obeys the same law only if the external field is uniform. Equation (3) shows in addition that even in the case of zero external field, the center-of-mass theorem fails if the particles have active gravitational mass too large to be neglected.

Astronomical data describe multiparticle systems. Milgrom therefore assumed explicitly that law (1) applies to the center of mass. But the examples above show that if law (1) applies to individual particles, it cannot in general apply to the center of mass. It can, however, apply to the center of mass in the special case of a system of one or more test particles (particles having negligible active mass) moving in a uniform external field. The galaxy systems studied by Milgrom are in general bound by their own active mass, and it is not altogether clear that a test-particle approximation is justified. If we simply postulate with Milgrom that law (1) applies generally to the center of mass, then it cannot apply in general to individual particles.

Milgrom dynamics is therefore incomplete at present and gives no clear prescription for particle motions. Naive applications require extreme

caution. More or less extensive changes in many-body dynamics are implied and cannot be predicted without a more complete theory. At present Milgrom's law must be thought of as a phenomenological modification of Kepler's laws rather than a systematic modification of Newtonian dynamics. Milgrom (1983a) is aware of drawbacks in the theory and discussed at some length another problem, namely an apparent violation of the principle of equivalence (A freely falling elevator is not equivalent to an unaccelerated inertial frame). A more complete theory is in course of development (Bekenstein and Milgrom 1983).

### III. COSMOLOGICAL IMPLICATIONS

Despite these difficulties, it is of some interest to speculate on the cosmological consequences of a Milgrom-type dynamics. Consider the classical Friedmann universes (Friedmann 1922; Rindler 1977, §§ 9.9 - 9.11), i. e., relativistic universes of zero pressure, and set the cosmological constant  $\Lambda = 0$ . The classical "closure parameter" is

$$\Omega \equiv \frac{\rho}{\rho_{c1}} = \frac{8\pi G\rho}{3H_0^2}, \quad (8)$$

where  $\rho$  is the universal mass density. If  $\Omega < 1$ , the universe is open and expands forever. If law (1) holds, Milgrom shows that most, if not all, galaxy velocity data can be interpreted with mass-to-luminosity ratios for galaxies equal to roughly 1 to 10 in solar units. (This M/L ratio measures all forms of mass which are clumped with the galaxies; uniformly distributed mass is, as always, excluded, but there is no evidence for its presence.) A ratio M/L  $\sim$  2 to 7 is roughly characteristic of stellar



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matter (Faber and Gallagher 1979), so there is no longer strong evidence for substantial quantities of mass other than that contained in stars. The mean  $M/L$  required to give  $\Omega = 1$  (Davis et al. 1980, Felten 1977) is much larger,  $\sim 1400 \ h$ . We conclude that  $\Omega \ll 1$ .

However, we cannot conclude that the universe will expand forever, because a Milgrom acceleration implies extensive changes in cosmology. To show this, I adapt a familiar argument (McCrea and Milne 1934; Bondi 1960, §§ 9.1-9.3; Peebles 1971, § 1b) from "Newtonian cosmology". Consider a comoving sphere of radius  $r(t)$  in the uniform cosmological fluid. Within a finite, spherically symmetric system, however large, Gauss's law holds in the Newtonian case and tells us that the acceleration of a point on the surface of the sphere depends only upon the mass within. We have

$$g_N = \ddot{r} = -\frac{G}{r^2} \frac{4\pi r_0^3 \rho}{3} = -\frac{\Omega_0^2 r_0^3}{2r^2}, \quad (9)$$

where  $\rho$  is the present density (a constant) and  $r_0$  is the present radius of our chosen sphere. Integration of this equation gives the exact Friedmann equation for the universal scale factor  $R(t)$ , from which we conclude that a zero-pressure universe with  $\Lambda = 0$  recollapses only for  $\Omega > 1$ .

It is easy to adapt this argument to a Milgrom acceleration, because equation (1) depends only upon  $g_N$ . In place of equation (9), we obtain, for Milgrom acceleration,

$$\ddot{r} = -\left(a_0 \frac{\Omega_0^2 r_0^3}{2r^2}\right)^{1/2}, \quad (10)$$

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assuming for the moment that  $|\ddot{r}| \ll \underline{a}_0$ , so that equation (1) applies. Integrating equation (10) once, we find that

$$(\dot{r})^2 = (\dot{r})_0^2 - (2\Omega H_0^2 a_0 r_0^3)^{1/2} \ln(r/r_0) . \quad (11)$$

For any assumed value of the present expansion velocity  $(\dot{r})_0$ ,  $\dot{r}(t)$  necessarily has a zero, i. e., there is a maximum radius and a turnaround. Introducing the usual notation  $\underline{r} \equiv r_0 R$  and imposing the initial condition  $\underline{H}_0 = (\dot{r}/r)_0$ , we find that the scale factor at turnaround is

$$R_{\max} \equiv r_{\max} r_0^{-1} = \exp \mu^2 , \quad (12)$$

where

$$\mu \equiv \left( \frac{H_0^2 r_0}{2\Omega a_0} \right)^{1/4} . \quad (13)$$

Substituting  $\underline{z} \equiv R_{\max}/R$  and integrating again, we find that the turnaround occurs after a time

$$\Delta t \equiv t_{\max} - t_0 = \frac{\mu}{H_0} R_{\max} \int_1^{R_{\max}} \frac{dz}{z^2 (\ln z)^{1/2}} . \quad (14)$$

The integral is an incomplete gamma function  $\Gamma(1/2, \ln R_{\max})$ . We have  $\Gamma \sim 1$  for  $R_{\max} \gtrsim 2$ , and  $\Gamma \rightarrow \sqrt{\pi}$  for  $R_{\max} \gg 1$ . Therefore

$$\Delta t \sim \pi^{1/2} \mu H_0^{-1} R_{\max} \quad \text{if } R_{\max} \gg 1 . \quad (15)$$

Having carried the Milgrom constant  $\underline{a}_0$  along explicitly, let us now set  $\underline{a}_0 \sim \underline{c} H_0$  as in Milgrom's theory. For a cosmological turnaround we set  $\underline{r}_0 \sim \underline{r}_H \equiv \underline{c}/H_0$ . (Note that, for  $\Omega \ll 1$  and  $\underline{a}_0 \sim \underline{c} H_0$ , the acceleration (10) is and

remains  $\ll \underline{a}_0$  for all values of  $\underline{r}_0 \lesssim \underline{r}_H$ , so law (1) should in fact apply. We have neglected any corrections to law (1) associated with  $\underline{v} \rightarrow \underline{c}$ ; these corrections should not dominate for  $\underline{r}_0 \lesssim \underline{r}_H$  and should certainly be small for  $\underline{r}_0 \ll \underline{r}_H$ .) Then  $\mu \sim (2\Omega)^{-1/4}$ . The turnaround time depends on  $\Omega$ , i. e., on the assumed present density. For  $\Omega = 10^{-2}$  we find  $\mu \sim 2.7$ ,  $\underline{R}_{\max} \sim 1200$ , and  $\Delta t \sim 5 \times 10^{13}$  yr. For  $\Omega = 10^{-3}$  the numbers are  $\mu \sim 4.7$ ,  $\underline{R}_{\max} \sim 5 \times 10^9$ , and  $\Delta t \sim 4 \times 10^{20}$  yr. Thus it seems that Milgrom universes of all densities recollapse, although the time required is very long for  $\Omega \ll 1$ . This result is the cosmic analog of Milgrom's (1983a) observation that the effective Milgrom potential of a point mass is logarithmic at large distances.

This derivation is a fraud, although it points the way to further work. Note the presence of  $\underline{r}_0$  in equation (13). There should be no need to set  $\underline{r}_0$  equal to  $\underline{r}_H$  or to any other specific value. In the analogous Newtonian derivation,  $\underline{r}_0$  drops out as soon as the initial-value condition  $\underline{H}_0 = (\dot{\underline{r}}/\underline{r})_0$  is applied. The presence of  $\underline{r}_0$  in equation (13) shows that  $\underline{R}_{\max}$  and  $\Delta t$  are different for spheres of different initial size. Therefore the "universe" described by equations (10) - (15) does not admit a universal scale factor  $\underline{R}(\underline{t})$  and, by the theorem of Robertson and Walker (Rindler 1977, § 9.5), cannot be homogeneous and isotropic. To improve this situation we might entertain the obvious possibility, suggested by Milgrom (1983c), that the constant  $\underline{a}_0$  is variable on a cosmic time scale. For example, we might set  $\underline{a}_0 \sim \underline{c}\dot{\underline{r}}/\underline{r}$  instead of  $\underline{a}_0 \sim \underline{c}\underline{H}_0$ . It may be evident, however, that this does not help. The parameter  $\underline{r}_0$  still fails to drop out of the equations, and we are still stuck with a universe which does not satisfy the cosmological principle.

To understand this paradox we must think about the logical basis of "Newtonian cosmology". There is an extensive literature on this (e. g., Layzer 1954; McCrea 1954; Raychaudhuri 1979, §§ 2.1-2.3), centering on the objection that the Newtonian potential is infinite in an infinite sea of mass. Two schools of thought have emerged to explain the remarkable success of the Newtonian calculation. The first school (Heckmann and Schücking 1959; Bondi 1960, §§ 9.1-9.3; Rindler 1977, §§ 9.2, 9.8) points out that the Newtonian argument leading to equation (9) above can be carried out within any finite sphere of mass, however large, and that the solution in an infinite sea must be the limit of the finite-sphere solutions and must therefore be the same. One might object that an infinite sea is also the limit of a finite cube or ellipsoid, and that these shapes certainly will give different solutions. The answer is that these solutions, unlike that in the spherical case, are not homogeneous and isotropic. It is argued that requiring the cosmological principle to be satisfied makes the large but finite sphere a unique and correct Newtonian model for cosmology. Thus in "Newtonian cosmology" the cosmological principle plays the role of, or replaces, a boundary condition. This justification cannot be extended to the "Milgrom universe" developed in equations (10)-(15). These equations describe a large, finite spherical universe, but observers within this sphere do not find its motion isotropic and homogeneous. It appears that a solution for a finite sphere satisfying the cosmological principle and equation (1) cannot be obtained.

The second school of thought on "Newtonian cosmology" (Callan, Dicke, and Peebles 1965; Peebles 1971, § Ib; Weinberg 1972, § 15.1) appeals to Birkhoff's theorem in general relativity (Lemaitre 1931, Bonnor 1962).

Birkhoff's theorem, the relativistic analog of Gauss's law, implies that the four-dimensional curvature is zero inside a concentric spherical cavity in a spherically symmetric mass distribution. This means that the gravitational field at the surface of a uniform sphere depends only upon the mass within the sphere and may be calculated by the Newtonian approximation when the sphere is small enough. The well-known weak-field linearity of general relativity is used implicitly in reaching this conclusion. This argument provides the strongest justification for the Newtonian calculation, although it is not a Newtonian argument. Once again we find that the argument breaks down when applied to a quasi-Newtonian calculation with a Milgrom acceleration. In a "Milgrom universe", a new theory of gravitation will have to be found, which reduces to a Milgrom acceleration in certain limits, for general relativity does not. Birkhoff's theorem may not be valid in such a theory of gravitation. Weak-field linearity will certainly fail, because equation (1) is nonlinear, as noted earlier.

The failure of Birkhoff's theorem would not be a fatal objection to a gravitational theory; in fact some might see it as an advantage. Birkhoff's theorem, which in effect limits the connection between local and global phenomena, limits general relativity's ability to account for numerical coincidences in cosmology. Relationships such as Milgrom's  $a_0 \sim cH_0$  might more readily be explained in the absence of Birkhoff's theorem. It is clear, however, that cosmology will be in a state of confusion if a Milgrom-type acceleration is verified. Naive intuition suggests that all universes may indeed recollapse because of the long-range character of the force, but we do not know how to prove this. A new theory of gravitation

will be needed. This difficult task should perhaps not be undertaken unless there is clear evidence that the Milgrom acceleration law holds on the scale of galaxies. Dressler and Lecar (1983) suggest that Milgrom's law does not explain the velocity data adequately.

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